Exercise 5D: Solutions

1 There is a total of 7 coins, of which a group of 4 are alike and another group of 3 are alike. These can be arranged in $\frac{7!}{4! \cdot 3!} = 35$ ways.

2 There is a total of 11 letters. Of these letters, 1 is M, 4 are S, 4 are I, and 2 are P. The 11 letters can therefore be arranged in $\frac{11!}{4! \cdot 4! \cdot 2!} = 34650$ ways.

3 There is a total of 11 letters. Of these letters, 2 are A, 2 are R, and 2 are O. The 11 letters can therefore be arranged in $\frac{11!}{2! \cdot 2! \cdot 2!} = 4989600$ ways.

4 There is a total of 8 digits. Of these digits, 5 are nine, and 3 are seven. Therefore, the 8 digits can be arranged in $\frac{8!}{5! \cdot 3!} = 56$ ways.

5 There is a total of 12 letters. Of these letters, 3 are A, 4 are B and 5 are C. Therefore, the 12 letters can be arranged in $\frac{12!}{3! \cdot 4! \cdot 5!} = 27720$ ways.

6 **a** There is a total of 8 flags. Of these, 2 are red, 2 are black, and 4 are blue. These can be arranged in $\frac{8!}{2! \cdot 2! \cdot 4!} = 420$ ways.

b The first flag is red so now there are 7 flags to arrange: 1 red, 2 black and 4 blue. This can be done in $\frac{7!}{2! \cdot 4!} = 105$ ways.

c The first and last flags are blue so so there are 6 flags to arrange: 2 red, 2 black and 2 blue. These can be arranged in $\frac{6!}{2! \cdot 2! \cdot 2!} = 90$ ways.

d Every alternate flag is blue. There are 4 flags to arrange in-between the blue flags: 2 red and 2 black. These can be arranged in $\frac{4!}{2! \cdot 2!} = 6$ ways. However we must multiply this by 2 as the blue flags can be placed in either odd or even positions. This gives a total of $6 \times 2 = 12$ arrangements.

e We group the two red flags together as one item. We then need to arrange this single group as well as 2 black flags and 4 blue flags. This makes 7 items in total. These can be arranged in $\frac{7!}{2! \cdot 4!} = 105$ ways.

7 Each path from A to B can be described by four R's and three D's in some order. As there are seven letters in total, there are $\frac{7!}{3! \cdot 4!} = 35$ paths.

8 **a** Let N be a movement of one unit in the north direction, and E be a movement of one unit in the east direction. Then each path from $(0, 0)$ to $(2, 4)$ can be described by two N's and four E's in some order. As there are six letters in total, there are $\frac{6!}{2! \cdot 4!} = 15$ paths.

b Let E be a movement of one unit in the east direction, and N be a movement of one unit in the north direction. Then each path from $(0, 0)$ to (m, n) can be described by m letter N's and n letter E's in some order. As there are $m + n$ letters in total, there are $\frac{(m+n)!}{m! \cdot n!}$ paths.

9 **a** The 52 playing cards can be arranged in $52!$ ways.

b There are now 104 playing cards, however each of the 52 playing cards occurs twice. Therefore, these cards can be arranged in $\frac{104!}{(2!)^{52}}$ ways.

c If there are n decks then there are $52n$ playing cards. Each of the 52 playing cards occurs n times. Therefore, these cards can be arranged in $\frac{(52n)!}{(n!)^{52}}$ ways.

10 Let N,E,S,W denote a movement of one unit in the north, east, south and west directions respectively. Any path of length 8 can be described by a combination of 8 of these letters in some order. Any path that starts at $(0, 0)$ and finishes at $(0, 0)$ has an equal number of N's and S's and an equal number of E's and W's. We list the various ways in which this can happen.

N	E	S	W	arrangements
0	0	4	4	$\frac{8!}{4! \cdot 4!} = 70$
1	1	3	3	$\frac{8!}{3! \cdot 3!} = 1120$
2	2	2	2	$\frac{8!}{2! \cdot 2! \cdot 2! \cdot 2!} = 2520$
3	3	1	1	$\frac{8!}{3! \cdot 3!} = 1120$
4	4	0	0	$\frac{8!}{4! \cdot 4!} = 70$

This gives a total of $70 + 1120 + 2520 + 1120 + 70 = 4900$.

11 There are various ways to solve this problem. We show one solution using arrangements. Jessica can either take 2 steps or 1 step at a time. Therefore, each of Jessica's paths up the stairs can be described by some sequence of 2's and 1's whose sum is 10. We list the various ways that this can happen.

digits	arrangements
1111111111	$\frac{10!}{10!} = 1$
111111112	$\frac{9!}{8!} = 9$
11111122	$\frac{8!}{6!2!} = 28$
1111222	$\frac{7!}{4!3!} = 35$
112222	$\frac{6!}{2!4!} = 15$
22222	$\frac{5!}{5!} = 1$

This gives a total of 89.